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Magnetically mediated superconductivity

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Abstract
We compare predictions of the mean-field theory of superconductivity for nearly antiferromagnetic and nearly ferromagnetic metals for cubic and tetragonal lattices. The calculations are based on the parametrization of an effective interaction arising from the exchange of the magnetic fluctuations and assume that a single band is relevant for superconductivity. We present intuitive arguments for why quasi-two-dimensional d-wave pairing is a particularly favourable case.

1. Introduction

In a series of papers [1–3], we have examined within a unified framework the possibility of superconductivity on the border of magnetic long-range order, assuming that the dominant interaction channel is of magnetic origin and depends on the relative spin orientations of the interacting quasiparticles. In order to understand certain qualitative features of the model and in the hope of providing some useful guidelines to experimentalists in the field, we have studied the role of the lattice structure (tetragonal versus cubic), whether one is on the border of ferromagnetic or commensurate antiferromagnetic long-range order. In this paper, we focus on the intuitive arguments that explain the results of the numerical calculations.

2. Model

We assume that a single band is relevant for superconductivity and consider quasiparticles in a simple tetragonal lattice described by a tight-binding dispersion relation

$$\epsilon_p = -2t(\cos(\mathrm{p}_x) + \cos(\mathrm{p}_y) + \alpha_t \cos(\mathrm{p}_z)) - 4t'(\cos(\mathrm{p}_x)\cos(\mathrm{p}_y) + \alpha_t \cos(\mathrm{p}_x)\cos(\mathrm{p}_z) + \alpha_t \cos(\mathrm{p}_y)\cos(\mathrm{p}_z)) \quad (1)$$

with hopping matrix elements t and t'. α_t models the electronic structure anisotropy along the z direction. $\alpha_t = 0$ corresponds to the quasi-2D limit while $\alpha_t = 1$ corresponds to the 3D cubic lattice. For simplicity, we measure all lengths in units of the respective lattice spacing.

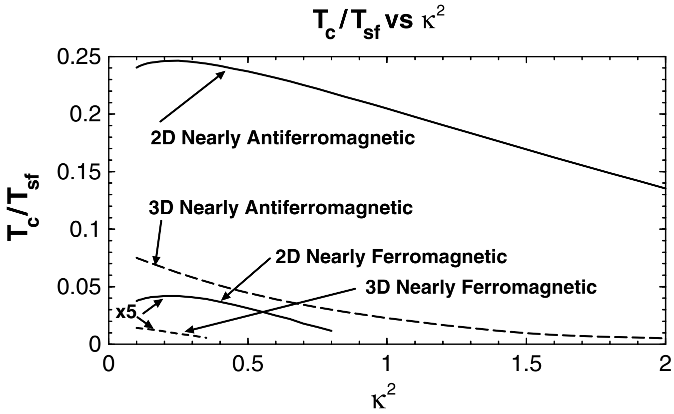

Figure 1. Eliashberg transition temperature for $T_{sf} = 0.67t$, $\kappa_0^2 = 12$, a physically realistic coupling constant $g^2\chi_0/t = 5$ as a function of the inverse correlation length κ for nearly antiferromagnetic and ferromagnetic systems in two and three dimensions. The transition temperatures for the nearly ferromagnetic case have been multiplied by a factor of five.

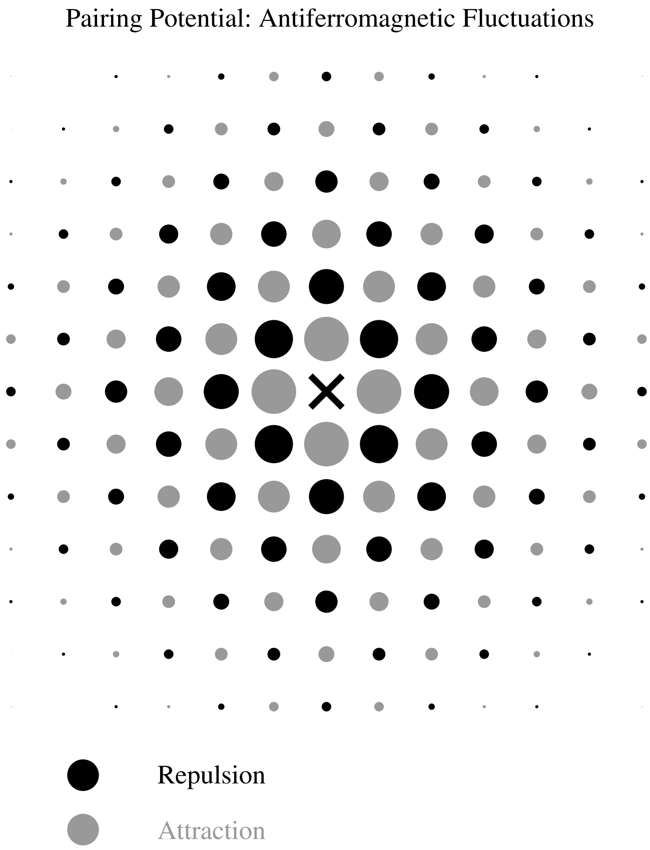

Figure 2. Static limit of pairing potential for antiferromagnetic fluctuations seen by the second quasiparticle given that the first is at the origin (marked by a cross).

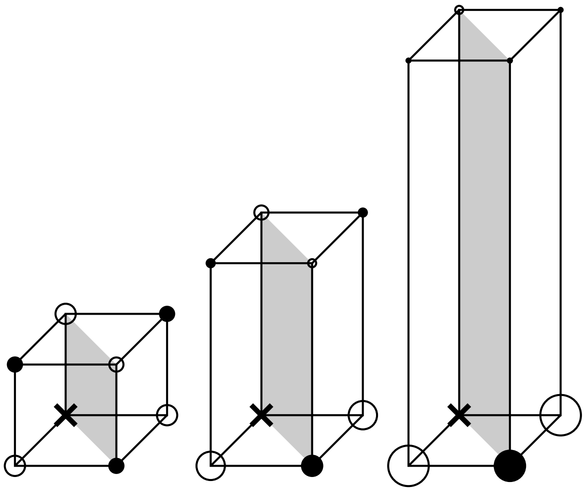

Figure 3. The pairing potential seen by a quasiparticle in a spin-singlet $d_{x^2-y^2}$ Cooper pair state given that the other quasiparticle is at the origin (marked by a cross). The figure shows the evolution of the potential as one goes from a cubic to a tetragonal lattice by varying the parameter α_m. Closed circles denote repulsive sites and open circles attractive ones. The size of the circle is a measure of the strength of the interaction. The nodal planes of the $d_{x^2-y^2}$ state are represented by the shaded regions.

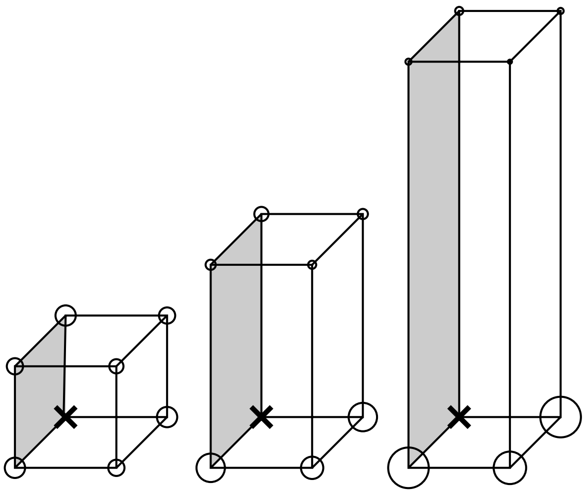

Figure 4. The magnetic potential seen by a quasiparticle in a spin-triplet p_x Cooper pair state given that the other quasiparticle is at the origin (marked by a cross). The figure shows the evolution of the potential as one goes from a cubic to a tetragonal lattice by varying the parameter α_m. Open circles denote attractive sites. The size of the circle is a measure of the strength of the interaction. The nodal planes of the p_x state are represented by the shaded regions.

The results shown below are all for a nearest-neighbour hopping $t' = 0.45t$ and a band filling factor $n = 1.1$.

The effective interaction between quasiparticles is assumed to be isotropic in spin space and is defined in terms of the coupling constant g and the generalized magnetic susceptibility, which is assumed to have a simple analytical form consistent with the symmetry of the lattice.

$$\chi(\boldsymbol{q}, \omega) = \frac{\chi_0 \kappa_0^2}{\kappa^2 + \hat{q}^2 - \mathrm{i}\frac{\omega}{\eta(\hat{q})}} \tag{2}$$

where κ and κ_0 are the correlation wavevectors or inverse correlation lengths in units of the lattice spacing in the basal plane, with and without strong magnetic correlations, respectively. Define

$$\hat{q}_{\pm}^2 = (4 + 2\alpha_m) \pm 2(\cos(q_x) + \cos(q_y) + \alpha_m \cos(q_z)) \tag{3}$$

where α_m parametrizes the magnetic anisotropy. $\alpha_m = 0$ corresponds to quasi-2D magnetic correlations and $\alpha_m = 1$ corresponds to 3D magnetic correlations.

In the case of a nearly antiferromagnetic metal, the parameters $\hat{q}^2$ and $\eta(\hat{q})$ in equation (2) are defined as

$$\hat{q}^2 = \hat{q}_+^2 \tag{4}$$

$$\eta(\hat{q}) = T_{sf}\hat{q}_-. \tag{5}$$

In the case of a nearly ferromagnetic metal the parameters $\hat{q}^2$ and $\eta(\hat{q})$ in equation (2) are defined as

$$\hat{q}^2 = \hat{q}_-^2 \tag{6}$$

$$\eta(\hat{q}) = T_{sf}\hat{q}_- \tag{7}$$

where T_{sf} is a characteristic spin fluctuation temperature.

3. Results

The reader is referred to the original papers [1–3] for details of the procedure used in the numerical solution of the Eliashberg equations for the transition temperature. One obtains a transition to a spin-triplet p-wave superconducting state in the case of nearly ferromagnetic systems and a spin-singlet $\mathrm{d}_{x^2-y^2}$ state in the nearly antiferromagnetic case.

Figure 1 shows the ratio T_c/T_{sf} as a function of κ^2. We remind the reader that, in our model, magnetic long-range order is at $\kappa = 0$. The first striking observation is that magnetic pairing is more robust for nearly antiferromagnetic systems than for ferromagnetic ones, given otherwise similar conditions. The second important trend one notices is that magnetic pairing is more favourable in quasi-two-dimensional than three-dimensional systems. The latter results are in qualitative agreement with the findings of [4, 5].

4. Discussion

In the case of magnetic pairing on the border of ferromagnetic long-range order, the pairing interaction is purely attractive in the spin-triplet channel, while in the case of spin-singlet pairing in nearly antiferromagnetic systems it is oscillatory in space and thus has both attractive and repulsive regions. One could therefore have expected that magnetic pairing mediated by ferromagnetic spin-fluctuations would have turned out to be the most favourable case. As pointed out in [1], on the border of ferromagnetism, magnetic pairing in the spin-triplet state has the disadvantage that only the exchange of magnetic fluctuations polarized along the direction of the interacting spins, i.e. longitudinal fluctuations, contributes to the quasiparticle interactions. For a spin-rotationally invariant system, both longitudinal and transverse fluctuations contribute to pairing only for a spin-singlet state.

For nearly antiferromagnetic metals, superconductivity depends on the ability of the electrons in a Cooper pair state to sample mainly the attractive regions of the oscillatory pairing potential. The fact that this turns out to be possible for quasi-two-dimensional tetragonal systems is illustrated in figure 2.

Figure 2 shows that in this case most of the repulsive regions of the pairing potential are located along the diagonal. Hence by selecting the Cooper pair state to vanish for $x = \pm y$, most of the repulsive regions are rendered harmless, and one can easily convince oneself that this is the only alternative. The magnetic pairing model successfully predicted the $d_{x^2-y^2}$ symmetry of the pairing state [6] for the cuprate superconductors.

The role played by lattice anisotropy is best explained by looking at the evolution of the static limit of the pairing potential as the lattice is made more and more anisotropic. The case of the nearly antiferromagnetic metal is shown in figure 3. The oscillatory pattern of the pairing potential is now three-dimensional and it is no longer possible to cancel out the repulsive regions of the quasiparticle interaction as effectively as in the quasi-two-dimensional case. Figure 3 shows that the strength of the interaction in the repulsive sites outside of the nodal plane of the $d_{x^2-y^2}$ state is reduced while at the same time the attraction in the basal plane is enhanced as one goes from the cubic to a more and more anisotropic tetragonal lattice. This enhancement is the consequence of the increase of the phase space of soft magnetic fluctuations as one goes from a cubic to a quasi-two-dimensional structure. Moreover, since our model potential varies smoothly with the tetragonal distortion, parametrized by α_m, the calculations reported in [3] show that these effects occur gradually with increasing separation between the basal planes.

The situation in the ferromagnetic case is depicted in figure 4. The interaction is attractive for all neighbouring sites of the origin. The increase of the phase space of soft magnetic fluctuations is obviously also at work in this case and the attraction in the basal plane is enhanced as one goes from the cubic to a more and more anisotropic tetragonal lattice. The calculations reported in [3] also show that these effects occur gradually with increasing separation between the basal planes for the same reason they do in the nearly antiferromagnetic case, namely the smooth dependence of the effective quasiparticle interaction on the anisotropy parameter α_m.

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